17th Annual
Regional High School Mathematics Contest

Sample Questions:

1. In a psychological experiment, 100 people are invited to a party. Each individual's arch-nemesis is also on the guest list. At the party each guest shakes hands with everyone else except for their nemesis. How many handshakes take place?

2. Find the area of quadrilateral $ABCD$ with vertices $(1, 3)$, $(5, 5)$, $(3, 1)$, and $(-1, -1)$.

3. Assume that
$$x + y + z = 1, \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0.$$
What is the value of $x^2 + y^2 + z^2$?

4. Let $\phi = \frac{1+\sqrt{5}}{2}$. Simplify $\frac{\phi^3 + \phi + 1}{\phi^5}$.

5. Assume that $y = ax^2 + bx + c$ passes through $(0, 1)$ and intersects the x-axis only at $(-2, 0)$. Find a, b, and c.

6. Let n be a positive even integer. Find the minimum of
$$f(x) = |x - 1| + |x - 2| + \cdots + |x - n|$$
in terms of n.

7. If $f(x) = x^4 + px^2 + qx + a^2$ is divisible by $x^2 - 1$, find the values of $f(a)$.

8. Suppose $\triangle ABC$ is a triangle with area 24 and that there is a point P inside $\triangle ABC$ which is at distance 2 from each side of $\triangle ABC$. What is the perimeter of $\triangle ABC$?

9. Define $a_1 = 2$ and
$$a_{n+1} = 1 - \frac{1}{a_n}, \quad \text{for } n = 1, 2, \ldots$$
Find $a_1 + a_2 + \cdots + a_{2015} + a_{2016}$.

10. Let $0 < \theta < \pi/2$ and $\frac{\cos(3\theta)}{\cos\theta} = \frac{1}{3}$. Find $\frac{\sin(3\theta)}{\sin\theta}$.

11. Let $0 \le a < b$. If
$$f(x) = -\frac{1}{2}x^2 + \frac{13}{2}$$
has the maximum value of $2b$ and the minimum value $2a$ on $[a, b]$, find the interval $[a, b]$.

12. You randomly select two distinct integers from 1 to 10. What is the probability that they are consecutive numbers, i.e. that they differ by 1?

13. What is the product of all real roots in the equation $x^{\log_{10} x} = 10$?

14. What is the radius of a circle inscribed in a rhombus with diagonals of length 10 and 24?

15. Suppose that a large triangle in the picture is equilateral with area 1. What is the area of the black region? Assume the black triangles for an infinitely nested pattern.