The Twenty-sixth Annual CNU Regional High School Mathematics Contest

February 14, 2026
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1. What is the ones digit of the base ten expansion of the number 7^{2026}?

   (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

2. The expression

$$\frac{2^1 + 2^0 + 2^{-1}}{2^{-2} + 2^{-3} + 2^{-4}}$$

   equals

   (A) 6 (B) 8 (C) $\frac{31}{2}$ (D) 24 (E) 512

3. Find the value of $(\log_3 2)(\log_4 3)\cdots(\log_{n+1} n)\cdots(\log_{32} 31)$.

   (A) 0.2 (B) 0.4 (C) 0.5 (D) 2 (E) 5

4. If $x^2 - 4x + c = 0$ has two equal real solutions, then the value of c is

   (A) -16 (B) -4 (C) 4 (D) 16 (E) None of these

5. If $\sin x + \cos x = \frac{1}{2}$, then $\sin x \cos x =$

   (A) $-\frac{3}{4}$ (B) $-\frac{3}{8}$ (C) $-\frac{1}{4}$ (D) $\frac{1}{4}$ (E) $\frac{5}{8}$

6. For how many real numbers x is $\sqrt{-(x+1)^2}$ a real number?

   (A) none (B) one (C) two

   (D) a finite number greater than two (E) infinitely many

7. Consider the sequence $c_1 = 1$, $c_2 = 8$, $c_3 = 21$, and so on. Each term is given by $c_n = an^2 + bn$. What is ab?

   (A) -6 (B) 0 (C) 1 (D) 6 (E) 8

8. If you have an unlimited supply of 5 cent and 11 cent postage stamps, what is the largest postage you cannot make up exactly?

   (A) 29 (B) 34 (C) 38 (D) 39 (E) 40

9. At exactly 3:40, what is the degree measure of the smaller angle formed by the hour hand and the minute hand of an analog clock?

   (A) 120° (B) 130° (C) 140° (D) 150° (E) None of these

10. Let $P(x)$ be a polynomial with integer coefficients. If $P(1) = 5$, which of the following could be the value of $P(5)$?

(A) 15 (B) 16 (C) 17 (D) 18 (E) 19

11. A linear function $f(x)$ has the following properties:

- the function f has a zero at $x = -4$, and
- for all real numbers x, $f(x + 2) = f(x) - 6$.

The equation of f in slope intercept form is:

(A) $y = -3x - 4$ (B) $y = -3x-12$ (C) $y = 3x + 4$ (D) $y = 3x + 12$ (E) Not uniquely determined

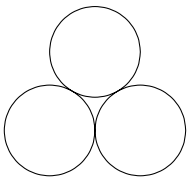
12. How many digits are in the base ten representation of the least common multiple of the first twelve positive integers?

(A) 5 (B) 7 (C) 9 (D) 11 (D) None of these

13. Two numbers x and y are chosen independently and uniformly at random from the interval $[0, 1]$. What is the probability that $x^2 + y^2 \leq 1$?

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{3}{4}$ (D) $\frac{\pi}{4}$ (E) None of these

14. Three congruent circles of radius 5 are stacked as pictured. What is the height of the stack?



(A) $5(1 + \sqrt{3})$ (B) 15 (C) $5(2 + \sqrt{3})$

(D) 20 (E) $10(1 + \sqrt{3})$

15. Suppose $b > 1$ and $x > 0$ and $(4x)^{\log_b(4)} - (5x)^{\log_b(5)} = 0$. Then x is:

(A) $\frac{1}{20}$ (B) $\frac{4}{5}$ (C) $\frac{5}{4}$ (D) 20 (E) Not uniquely determined

16. ΔABC has sides AB of length 6, BC of length 3, and AC of length 4. The angle bisector of $\angle C$ intersects side AB in point D What is the length of the segment AD?

(A) $\frac{18}{7}$ (B) $\frac{24}{7}$ (C) $\frac{18}{5}$ (D) 3 (E) 4

17. If the first four terms of an arithmetic sequence are a, x, b, $2x$, then the fraction $\frac{b}{a}$ equals

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 2 (D) 3 (E) 4

18. Let $P(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial. Which of these expressions gives the sum of the odd-indexed coefficients of $P(x)$?

(A) $P(1)$ (B) $P(1) + P(-1)$ (C) $P(1) - P(-1)$

(D) $\dfrac{P(1) + P(-1)}{2}$ (E) $\dfrac{P(1) - P(-1)}{2}$

19. Let c be a constant. The system of equations

$$\begin{aligned} x - y &= 2 \\ cx + y &= 3 \end{aligned}$$

has a solution in the first quadrant if and only if

(A) $c = -1$ (B) $c > -1$ (C) $c < \dfrac{3}{2}$ (D) $0 < c < \dfrac{3}{2}$ (E) $-1 < c < \dfrac{3}{2}$

20. Let $\lfloor z \rfloor$ denote the greatest integer not exceeding z. Let x and y satisfy the simultaneous equations

$$\begin{aligned} y &= 7\lfloor x \rfloor + 8 \\ y &= 3\lfloor x - 2 \rfloor - 2\,. \end{aligned}$$

If x is not an integer, then $x + y$ is

(A) between -25 and -24 (B) between -24 and -23 (C) between -5 and -4

(D) between -4 and -3 (E) an integer

21. Consider an octagon O with side length s. For what value of s does the area of O equal half of its perimeter?

(A) $\dfrac{1}{4}(\sqrt{2} - 1)$ (B) $\dfrac{1}{2}(\sqrt{2} - 1)$ (C) $\sqrt{2} - 1$ (D) $2(\sqrt{2} - 1)$ (E) $4(\sqrt{2} - 1)$

22. Let a, b, c be distinct positive numbers. If $a^x - b^x + c^x > 0$ holds if and only if $x < 1$, then

(A) $a + c > 2b$ (B) $a + c < 2b$ (C) $e^a + e^c > 2e^b$

(D) $1/a + 1/c < 2/b$ (E) None of the these

23. A triangle has vertex A at $(0, 3)$, vertex B at $(4, 0)$, and vertex C at $(x, 5)$ for some x between 0 and 4. If the area of the triangle is 8, then what is the value of x?

(A) 2 (B) $\dfrac{7}{3}$ (C) $\dfrac{8}{3}$ (D) 3 (E) $\dfrac{10}{3}$

24. If $i = \sqrt{-1}$, then the sum of

$$\sin\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}+\frac{\pi}{2}\right) + \cdots + i^n \sin\left(\frac{\pi}{4}+n\cdot\frac{\pi}{2}\right) + \cdots + i^{2026}\sin\left(\frac{\pi}{4}+2026\cdot\frac{\pi}{2}\right)$$

equals

(A) $\frac{\sqrt{2}}{2}$ (B) $-1013\sqrt{2}$ (C) $\frac{1014\sqrt{2}}{2}$

(D) $\frac{\sqrt{2}}{2}(1013+1013i)$ (E) $\frac{\sqrt{2}}{2}(1014+1013i)$

25. Let a and b be positive real numbers. How many real solutions for x are there to the following equation?

$$\frac{x-a}{b}+\frac{x-b}{a} = \frac{b}{x-a}+\frac{a}{x-b}$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) Not enough information

26. The product

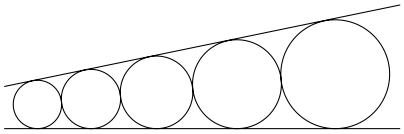
$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{9^2}\right)\left(1-\frac{1}{10^2}\right)$$

equals

(A) $\frac{5}{12}$ (B) $\frac{1}{2}$ (C) $\frac{11}{20}$ (D) $\frac{2}{3}$ (E) $\frac{7}{10}$

27. In the figure below, the five circles are tangent to one another consecutively and to the lines L_1 and L_2. If the radius of the largest circle is 18, and the radius of the smallest is 8, what is the radius of the middle circle?

(A) 12 (B) 12.5 (C) 13 (D) 13.5 (E) 14



28. Find the sum of all possible integers k such that $1+2+\cdots+k = n^2$ for some integer $1 < n < 100$.

(A) 34 (B) 41 (C) 48 (D) 53 (E) 57

29. Let S be a square of side length 1. What is the greatest value of x such that five squares of side length x can be packed into S without overlapping?

(A) $\frac{4-\sqrt{2}}{7}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2\sqrt{2}-1}{7}$ (D) $\frac{2-\sqrt{2}}{2}$ (E) $\frac{3-\sqrt{2}}{4}$

30. Let $\sin\alpha+\sin\beta+\sin\gamma = \cos\alpha+\cos\beta+\cos\gamma$. Find $\cos^2\alpha+\cos^2\beta+\cos^2\gamma$.

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 3