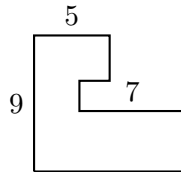


TEAM ROUND ANSWER KEY

1. If $\frac{5}{12} = \frac{1}{a} + \frac{1}{b}$ and $ab = 24$, find $a + b$

Answer: 10

2. In the following figure, every angle is a right angle. Find the perimeter of the figure.



Answer: 42

3. John stands on the left bank of a river. There are nine stepping stones that form a path leading across the river to the right bank. John will cross the river using these stones, either **stepping** directly to the next stone in the path or skipping a stone and **hopping** one beyond it. John always moves toward the right bank. How many different ways are there in total for John to journey to the right side of the river?

For example, from the left bank, John could hop to stone 2, step to stone 3, step to stone 4, hop to stone 6, step to stone 7, hop to stone 9, and then step to the right bank. That's one way for John to make the journey.

Answer: 89

4. How many positive cubes divide $(3!)(5!)(7!)(9!)$?

Answer: 36

5. Three fair ten-sided dice, with sides numbered 1–10, are tossed. What is the probability that the three numbers turned up can be arranged to form an arithmetic progression with common difference two? Write your answer as a fraction in reduced form.

Answer: $\frac{9}{250}$

6. Find an integer n such that $\sum_{i=1}^n (6i^2 + 2) = 340$

Answer: $n = 5$ (Simply writing "5" is acceptable.)

7. Mary drives from Allentown to Bethlehem at an average speed of 40 miles per hour. On the way back, she takes the same route but drives at an average speed of 60 miles per hour. What is her average speed (in miles per hour) for the entire round trip?

Answer: 48

8. Let A and B denote the points of intersection of the circles $x^2 + y^2 + 4x - 4y = 5$ and $x^2 + y^2 - 4x - 8y = 1$. Find the slope of the segment AB .

Answer: -2

9. Let $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. Find

$$\frac{\binom{8}{0}}{1} + \frac{\binom{8}{1}}{2} + \frac{\binom{8}{2}}{3} + \cdots + \frac{\binom{8}{8}}{9}.$$

Answer: $\frac{511}{9}$

10. The function $f(x) = \sin^2 x - \sin x + 2$ has a range of the form $[a, b]$ where a and b are real numbers with $a < b$. Find $a + b$.

Answer: $\frac{23}{4}$ or 5.75

11. Find a pair of positive, odd integers x and y so that $x + y = 316$ where x is divisible by 13 and y is divisible by 11. (Give your answer as an ordered pair (x, y) .)

Answer: (195,121)

12. A drawer in a darkened room contains 100 red socks, 80 green socks, 60 blue socks, and 40 black socks. A person selects socks one at a time from the drawer but is unable to see the color of the socks drawn. What is the smallest number of socks that must be selected to guarantee that the selection contains at least 10 pairs of socks? (A pair of socks is two socks of the same color. No sock may be counted in more than one pair.)

Answer: 23